The Basics of Set Theory

Set Theory

- Some atonal music is organized using twelve-tone rows, but this method was not developed until the 1920s
- Atonal music before this time was freely atonal, and more difficult for analysts to describe
- Modern music theorists have developed a way of describing this atonal music which we call **pitch-class set theory** (or just set theory)
- Set theory takes quite a while to figure out and requires some mathematical skills; but if you are interested in learning more about how atonal music is organized, it is a necessary skill to acquire

Pitch classes and interval classes

- *Pitch classes*
  - There are 12 pitch classes in our chromatic scale—these are numbered from 0 to 11
  - For example, C, B♯ and D𝄫 belong to the same *pitch class* which we call 0
- *Interval classes*
  - Similarly, an augmented unison and minor second belong to the same *interval class* (both have 1 semitone)
  - We only have interval classes 1 through 6—for larger intervals, we invert them, so 11 (M7) is the same as 1 (m2), 10 (m7) is the same as 2 (M2), etc.

Pitch class sets

- Set theory involves the **segmentation** of a piece of music into different groups of pitches called **pitch class sets** (or just **pc sets**)
  - We might group notes that are rhythmically close together, that are in the same gesture or phrase, are in the same register, have the same timbre, etc.
  - We name the set of pitches in the group and compare it to other sets of pitches, looking for patterns
- These pc sets are written by listing all of their pitches, separated by commas, enclosed in parentheses (B, G, G♯)
- Pitch class sets give us a way of describing any combination of pitches systematically

Set classes

- You can perform different operations on pc sets such as transposition, inversion, retrograde, reordering or permutation, verticalization, octave displacement, etc.
- All of these pitch class sets are related to each other—they all belong to the same **set class**
- It's kind of like a triad—we can invert a triad, space it out differently, transpose it, etc. and it will still be a triad
  - In fact, all major and minor triads belong to the same set class, which is called set class (0 1 5) or set class 3-4
- All pitch class sets related by transposition or inversion belong to the same set class

Normal order

- There are three steps involved in figuring out what ***set class*** a pitch class set belongs to.
- Step 1: Find the *normal order* of the pitches in the set
  - Arrange the pitches into their **most compact ascending order** to get the normal order
  - Compare the different ascending orderings and choose the one with the **smallest interval from bottom to top**
  - Brackets are used to indicate a pc set that is in normal order
    - Example: the normal order of pc set (B, G#, G) is [G, G#, B]
  - If there is a tie in this first step, choose the ordering that is **most compact to the left** (with the smallest intervals first)

Best normal order

- Step 2: Find the *best normal order* by comparing the normal order of the original set with the normal order of the inverted form of the set
  - In set theory, a set and its inversion are considered related
  - One way to invert a pc set is to convert the pitch names to pitch classes, and then subtract each pitch class from 12
    - Example: [G, G#, B] = [7, 8, 11]
    - (12 - 7 = 5; 12 - 8 = 4; 12 - 11 = 1)
    - The inverted set is [5, 4, 1]
  - Find the normal order of the inverted set = [1, 4, 5]
  - The **best normal order** is the version of the set that is **most compacted to the left** (original or inverted) = [7, 8, 11]

Prime form

- Step 3: Find the *prime form* (the form of the set that is used to represent all of the others)
  - There are as many as 24 pc sets that are related to a prime form—2 transpositions and their inversions—all of these belong to the same *set class*
  - To find the prime form, take the best normal order and begin it on 0, calculating the distance between pitches in semitones
  - The prime form is represented by numbers in parentheses with no commas
    - Example: the prime form of [G, G♯, B] or [7, 8, 11] is (0 1 4)
  - As if this weren't complicated enough, we generally refer to set classes not by their prime form, but by their Forte number (see Appendix C) (0 1 4) = set class 3-3